Attack and Release Time Constants in RMS-Based Feedback Compressors 4703 (P13-3)

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Attack and Release Time Constants in RMS-Based Feedback Compressors

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The goal of the paper is to develop mathematical models for feedback and feedforward compressors. A couple of possible configurations are explored: linear-output RMS detector with linearly-controlled VCA (linear-domain compressor) and logarithmic-output RMS detector with exponentially-controlled VCA (log-domain compressor). It is shown that the transfer functions of both configurations are equivalent. A formula for transforming the compression ratio of a log-domain compressor to that of a linear-domain compressor is derived. The differences between feedforward and feedback compressor configurations, with regard to time constants and performance, are considered.

0 Introduction

The core of any compressor consists of two elements: a level detector and an amplifier with variable gain. The output of the level detector is a *dc* current or voltage that is a representation of the *ac* input signal. The detector could be a peak detector, an average detector or a true Root Mean Square (RMS) detector. The peak detector is usually low cost and can be fast. The signal detected is the maximum level of the signal. The average detector computes the mean level of the signal. It is rather inexpensive and the time constants are comparable to the RMS detector ones. The true RMS detector is the only detector that directly relates to the power of the signal, independent of the signal waveform. The RMS value of an *ac* voltage or current is the equivalent *dc* voltage or current that generates the same amount of real power in a resistive load. If the shape of the incoming signal is known, e.g., sine wave, square wave, then either the peak or the average detector can be used to calculate the RMS level of the signal. Unfortunately, the waveform of a music source can not be predicted. This is the reason why the peak and average detector are not necessarily appropriate for audio applications. Any of the above detectors could be implemented in digital domain as well. For instance, the *ac* signal can be digitized by an analog to digital converter, or ADC, and the computation takes place in a digital signal processing unit, DSP, or micro controller. Another way is to digitize the *dc* output of a true RMS detector and use the microcontroller to control compression ratio. The compressors considered for this paper are based on true RMS detectors.

The variable gain amplifier is usually a three port device: input, output and gain control. Any of the ports could be voltage or current connections. The variable gain amplifier could be a voltage divider with variable shunt impedance (bipolar or JFET transistor, opto-resistor), an operational transconductance amplifier (OTA), a voltage controlled amplifier (VCA), a digitally controlled attenuator, or a multiplier in a DSP. The voltage divider and shunt impedance solution has the drawbacks of high distortion, limited dynamic range and unpredictable transfer function. The OTA has the advantage of a defined transfer function and a rather better dynamic range. Yet, the distortion performance of these devices is not appropriate for professional audio applications [1]. A digitally controlled attenuator is more difficult to implement since it needs a digitized value of the RMS level [5]. Another problem is that the attenuation steps are often large and the compressor can not be controlled in-between steps. Digitally controlled attenuators are also prone to zipper noise. The DSP based variable gain amplifier is essentially a multiplier and it makes sense in the context of a DSP-based compressor. In this case the variable gain element is linearly controlled. VCAs are the highest performance variable gain amplifiers [2] [3]. Recent developments in voltage controlled amplifier [4] show the commitment of the IC manufacturers to improving this popular device. The most popular VCAs have exponential gain control. The advantage is that the exponential transfer function is linear in decibels. In order to use these VCAs in compressor applications, the detector should preferably have a logarithmic output. In other words, the *dc* output of the detector is a logarithmic representation of the *ac* input signal.

1 RMS detector

The theory of the RMS detector was described in a previous paper [6]. The RMS value, V_{rms}, of an input signal $v_{in}(t)$ is defined as:

$$V_{rms} = \lim_{T\to\infty} \sqrt{\frac{1}{T} \int_{-\infty}^{T} v_{in}^2(t) \cdot dt} \tag{1}$$

As stated above, the RMS detector can have a linear or logarithmic output. The block diagram of each detector is shown in reference [6]. In a linear output detector, the input signal is squared, then integrated over a finite time and eventually the square root is extracted The problem with this approach is that the square operation needs a lot of dynamic range. For instance, to detect professional audio signals that could be as high as +24 dBu over 80dB dynamic range, the peak output voltage of the square block swings between 3.03 μV and 303 V. The dynamic range required at the output of the square block is 160 dB ! In the digital domain this dynamic range translates to 27 bit words. One way around this limitation is to compress the signal going into the linear RMS detector. Then, the output of the linear RMS detector is multiplied by the compression factor. The integrator is a first order low pass filter. The time constant of the integrator determines the ripple as well as the transient response at the detector output.

The logarithmic RMS detector is more forgiving in terms of dynamic range of voltage required. The input signal needs to be rectified because the logarithm function is defined only for positive values. Next step is to take the logarithm of the rectified signal. The square operation is just a multiplication by a factor of two. Thus, the dynamic range required at the output of the square block is reduced to 25.3 dB. The signal is integrated by a first order log filter described in reference [7]. A limiting factor can be the amount of current available for the log integrator. The current in the log filter diode is proportional to the square of the input current. Thus, the RMS detector based on log filter requires a large dynamic range in current instead of voltage.

2 Compressor topologies

2.1 Linear-Domain Compressor

2.1.1 Feedforward

The block diagram of a linear-domain feedforward compressor is shown in Fig. 1 . The RMS detector and VCA are linear devices. The input of the RMS detector is pre-scaled to a reference level, V_r . This voltage is also called *reference level*. At reference level, the output of the linear RMS detector is *one*. The voltage at the gain control port Gc is applied to an internal math block. The output voltage is the product of the input voltage multiplied by the gain control voltage. In the case of a linear VCA, used in a compressor application, the transfer function is:

$$f(G_c) = \frac{1}{G_c} \qquad (2)$$

The VCA output and input voltage have the same polarity.

The output of the linear RMS detector is calculated in reference [6] as follows:

$$V_{rms}(t) = \left[\int \frac{1}{\tau} \cdot \left(\frac{V_{in}(t)}{V_r}\right)^2 \cdot \exp\left(\frac{t}{\tau}\right) \cdot dt + c\right]^{\frac{1}{2}} \cdot \exp\left(-\frac{t}{2 \cdot \tau}\right) \qquad (3)$$

where τ is the time constant of the integrator and c is a constant without units.

The RMS detector output, V_{rms}, is applied to the gain control port Gc of the linear VCA. Assume that the multiplier factor k is one. The function of k factor is explained later in this paper. The transfer function of the feedforward compressor is calculated as follows:

$$\frac{V_{out}(t)}{V_{in}(t)} = f(V_{rms}) = \frac{1}{V_{rms}(t)} \qquad (4)$$

Substituting V_{rms} from equation (3) into equation (4) the transfer function becomes:

$$\frac{V_{out}(t)}{V_{in}(t)} = \left[\int \frac{1}{\tau} \cdot \frac{V_{in}^2(t)}{V_r^2} \cdot \exp\left(\frac{t}{\tau}\right) \cdot dt + c\right]^{-\frac{1}{2}} \cdot \exp\left(\frac{t}{2 \cdot \tau}\right) \qquad (5)$$

2.1.2 Feedback

The block diagram of a linear-domain feedback compressor is shown in Fig. 2. The RMS detector and VCA are also linear devices. The transfer function of the linearly controlled VCA is according to equation (2). The detector input is the output voltage divided by reference level V_r . Equation (3) can not be applied directly, as in the case of the feedforward compressor, because the output voltage is a function of the detector output, V_{rms} . Thus, it is necessary to go back one step to the differential equation that describes the functionality of the linear RMS detector. From reference [6], the differential equation of the linear RMS detector is calculated as follows:

$$\frac{V_{out}^2(t)}{V_R^2} = \tau \cdot \frac{\partial V_{rms}^2(t)}{\partial t} + V_{rms}^2(t) \tag{6}$$

Assuming that the multiplier factor k is one, the output voltage can be calculated from equation (4) as follows:

$$V_{out}(t) = \frac{V_{in}(t)}{V_{rms}(t)} \tag{7}$$

Substituting equation (7) into (6) and rearranging the terms, the differential equation can be written as:

$$\frac{V_{in}^2(t)}{V_r^2} = \tau \cdot V_{rms}^2(t) \cdot \frac{\partial V_{rms}^2(t)}{\partial t} + V_{rms}^4(t) \tag{8}$$

Noting that:

$$x \cdot \frac{\partial x}{\partial t} = \frac{1}{2} \cdot \frac{\partial x^2}{\partial t} \tag{9}$$

Applying the above property to equation (8) and rearranging the terms, the latter can be written as follows:

$$\frac{\partial V_{rms}^4(t)}{\partial t} + \frac{2}{\tau} \cdot V_{rms}^4(t) = \frac{V_{in}^2(t)}{V_r^2} \cdot \frac{2}{\tau} \tag{10}$$

The algorithm for solving the above differential equation is described in references [6] and [8]. Following the same steps as in reference [6], we make the following notations:

$$\begin{aligned} y(t) &= V_{rms}^4(t) \\ u(t) &= \frac{2 \cdot t}{\tau} \end{aligned} \tag{11}$$

Also, notice the following identity:

$$\frac{\partial}{\partial t}[y(t) \cdot \exp(u(t))] = \left[\frac{\partial y(t)}{\partial t} + \frac{2}{\tau} \cdot y(t)\right] \cdot \exp(u(t)) \tag{12}$$

Let's multiply the left and right terms of equation (10) by exp(u(t)) and use identity (12). The result follows:

$$\frac{\partial}{\partial t}[y(t) \cdot \exp(\tfrac{2 \cdot t}{\tau})] = \frac{V_{in}^2(t)}{V_r^2} \cdot \frac{2}{\tau} \cdot \exp(\tfrac{2 \cdot t}{\tau}) \tag{13}$$

Differential equation (13) can be solved by integrating both terms with respect to time:

$$y(t) = \left[\int \tfrac{2}{\tau} \cdot \frac{V_{in}^2(t)}{V_r^2} \cdot \exp(\tfrac{2\cdot t}{\tau}) \cdot dt + c \right] \cdot \exp(-\tfrac{2\cdot t}{\tau}) \tag{14}$$

and by substituting y(t) with its definition from (11), the output of the linear RMS detector, V_{rms}, has the following representation:

$$V_{rms}(t) = \left[\int \tfrac{2}{\tau} \cdot \frac{V_{in}^2(t)}{V_r^2} \cdot \exp(\tfrac{2\cdot t}{\tau}) \cdot dt + c \right]^{\frac{1}{4}} \cdot \exp\left(-\tfrac{t}{2\cdot\tau}\right) \tag{15}$$

where c is a constant without units.

Substituting equation (15) into (4), the transfer function of the feedback compressor can be calculated as follows:

$$\frac{V_{out}(t)}{V_{in}(t)} = \left[\int \tfrac{2}{\tau} \cdot \frac{V_{in}^2(t)}{V_r^2} \cdot \exp(\tfrac{2\cdot t}{\tau}) \cdot dt + c \right]^{-\frac{1}{4}} \cdot \exp\left(\tfrac{t}{2\cdot\tau}\right) \tag{16}$$

Notice that the time constant of the RMS detector integrator is halved by the feedback.

2.2 Log-Domain Compressor

2.2.1 Feedforward

The block diagram of a log-domain feedforward compressor is shown in Fig. 1. In this case, the RMS detector is logarithmic and the VCA is exponential. The RMS detector input voltage is pre-scaled to reference level V_r. At reference level, the output of the logarithmic detector is zero. An extra multiplier block is added between the output of the RMS detector and VCA gain control port. Multiplier k defines the compression ratio, which is analyzed later in this paper. Notice that factor k multiplies a logarithmic representation of the RMS detector input signal. A similar multiplier block, in the linear-domain compressor, was not shown because it does not have a direct and obvious correspondence with its counterpart in the log-domain compressor. In the case of the exponentially controlled VCA, the transfer function at the control port is [9]:

$$f(G_c) = \exp\left(-\tfrac{G_c}{V_T}\right) \tag{17}$$

where V_T is the thermal voltage equal to 0.0259 V at 27° C.

In reference [9] the voltage applied to the gain control port is divided by two which represents a square root operation. In real applications the square root operation in the logarithmic RMS detector is transferred to the VCA. In the case of the log-domain compressor, it doesn't matter where the division by two is made. So, in order to be consistent with the description of the linear-domain compressor, the square root operation is done in the RMS detector. The output of the logarithmic RMS detector is calculated in reference [6] as follows:

$$V_{rms}(t) = -\frac{V_T \cdot t}{2 \cdot \tau} + V_T \cdot \ln \sqrt{\int \frac{1}{\tau} \cdot \frac{V_{in}^2(t)}{V_r^2} \cdot \exp(\tfrac{t}{\tau}) \cdot dt + c} \tag{18}$$

where τ is the time constant of the log integrator [7] and c is a constant without units.

The transfer function of the feedforward compressor is calculated as follows:

$$\frac{V_{out}(t)}{V_{in}(t)} = f(k \cdot V_{rms}) = \exp\left(-\frac{k \cdot V_{rms}(t)}{V_T}\right) \tag{19}$$

Substituting equation (18) into equation (19), the transfer function becomes:

$$\frac{V_{out}(t)}{V_{in}(t)} = \exp\left(\frac{k \cdot t}{2 \cdot \tau} - k \cdot \ln \sqrt{\int \frac{1}{\tau} \cdot \frac{V_{in}^2(t)}{V_r^2} \cdot \exp(\tfrac{t}{\tau}) \cdot dt + c}\right) \tag{20}$$

Rearranging the terms in above equation, the transfer function has the following form:

$$\frac{V_{out}(t)}{V_{in}(t)} = \left[\int \frac{1}{\tau} \cdot \frac{V_{in}^2(t)}{V_r^2} \cdot \exp(\tfrac{t}{\tau}) \cdot dt + c\right]^{-\frac{k}{2}} \exp\left(\frac{k \cdot t}{2 \cdot \tau}\right) \tag{21}$$

Notice that for k = 1, equation (21) has the same expression as the transfer function for the linear-domain feedforward compressor.

2.2.2 Feedback

The block diagram of a log-domain feedback compressor is shown in Fig. 2. The RMS detector is logarithmic and the VCA is exponentially controlled. The transfer function at the control port of the exponentially controlled VCA is given by definition (17). As in the case of the linear-domain feedback compressor, the input of the RMS detector is proportional to the output voltage of the feedback compressor. The output voltage is calculated in equation (19) and is a function of the VCA gain which is a function of the output of the logarithmic RMS detector. Thus, the differential equation of the logarithmic RMS detector has to be solved for a different input signal. The differential equation of the logarithmic output RMS detector was calculated in reference [6] as follows:

$$\frac{V_{out}^2(t)}{V_r^2} = \tau \cdot \frac{\partial \exp\left(\frac{2 \cdot V_{rms}(t)}{V_T}\right)}{\partial t} + \exp\left(\frac{2 \cdot V_{rms}(t)}{V_T}\right) \tag{22}$$

The output voltage is calculated from equation (19) as follows:

$$V_{out}(t) = V_{in}(t) \cdot \exp\left(-\frac{k \cdot V_{rms}(t)}{V_T}\right) \tag{23}$$

Substituting equation (23) into equation (22) and rearranging terms, the RMS detector differential equation can be written as:

$$\frac{V_{in}^2(t)}{V_r^2} = \tau \cdot \exp\left(\frac{2 \cdot k \cdot V_{rms}(t)}{V_T}\right) \cdot \frac{\partial \exp\left(\frac{2 \cdot V_{rms}(t)}{V_T}\right)}{\partial t} + \exp\left(\frac{2 \cdot (k+1) \cdot V_{rms}(t)}{V_T}\right) \quad (24)$$

Notice that the following identities:

$$\frac{\partial \exp\left(\frac{2 \cdot V_{rms}(t)}{V_T}\right)}{\partial t} = \frac{1}{V_T} \cdot \exp\left(\frac{2 \cdot V_{rms}(t)}{V_T}\right) \cdot \frac{\partial (2 \cdot V_{rms}(t))}{\partial t} \quad (25)$$

$$\frac{\partial \exp\left(\frac{2 \cdot (k+1) V_{rms}(t)}{V_T}\right)}{\partial t} = \frac{k+1}{V_T} \cdot \exp\left(\frac{2 \cdot k \cdot V_{rms}(t)}{V_T}\right) \cdot \exp\left(\frac{2 \cdot V_{rms}(t)}{V_T}\right) \cdot \frac{\partial (2 \cdot V_{rms}(t))}{\partial t} \quad (26)$$

Using the above identities, equation (24) can be written as follows:

$$\frac{V_{in}^2(t)}{V_r^2} = \frac{\tau}{k+1} \cdot \frac{\partial \exp\left(\frac{2 \cdot (k+1) \cdot V_{rms}(t)}{V_T}\right)}{\partial t} + \exp\left(\frac{2 \cdot (k+1) \cdot V_{rms}(t)}{V_T}\right) \quad (27)$$

In order to solve the above differential equation, let's make the following notations:

$$z(t) = \exp\left(\frac{2 \cdot (k+1) \cdot V_{rms}(t)}{V_T}\right)$$
$$v(t) = \frac{(k+1) \cdot t}{\tau} \quad (28)$$

This differential equation is solved the same way as the differential equation for the linear output RMS detector. Substituting z(t) in equation (27) and multiplying both sides by exp(z(t)), the differential equation can be written as follows:

$$\frac{\partial}{\partial t}[z(t) \cdot \exp(v(t))] = \frac{k+1}{\tau} \cdot \frac{V_{in}^2(t)}{V_r^2} \cdot \exp(v(t)) \quad (29)$$

Both sides of equation (29) are integrated and substituting z(t) from (28) the following equation is obtained:

$$\exp\left(\frac{2 \cdot (k+1) \cdot V_{rms}(t)}{V_T}\right) = \left[\int \frac{k+1}{\tau} \cdot \frac{V_{in}^2(t)}{V_r^2} \cdot \exp\left(\frac{(k+1) \cdot t}{\tau}\right) \cdot dt + c\right] \cdot \exp\left(-\frac{(k+1) \cdot t}{\tau}\right) \quad (30)$$

where c is a constant without units.

Equation (30) is solved for $V_{rms}(t)$ as follows:

$$V_{rms}(t) = -\frac{V_T \cdot t}{2 \cdot \tau} + \frac{V_T}{k+1} \cdot \ln \sqrt{\int \frac{k+1}{\tau} \cdot \frac{V_{in}^2(t)}{V_r^2} \cdot \exp\left(\frac{(k+1) \cdot t}{\tau}\right) \cdot dt + c} \quad (31)$$

The transfer function of the feedback compressor is calculated by substituting equation (31) into equation (19) as follows:

$$\frac{V_{out}(t)}{V_{in}(t)} = \left[\int \frac{k+1}{\tau} \cdot \frac{V_{in}^2(t)}{V_r^2} \cdot \exp\left(\frac{(k+1)\cdot t}{\tau}\right) \cdot dt + c\right]^{-\frac{k}{2\cdot(k+1)}} \cdot \exp\left(\frac{k\cdot t}{2\cdot\tau}\right) \quad (32)$$

Notice that in the case of the feedback compressor the log integrator time constant is divided by the factor (k+1). For k = 1, the transfer function of the log-domain feedback compressor is identical to the transfer function of the linear-domain feedback compressor, equation (16).

3 Compression ratio

The compression ratio is defined as the rate of change at the input of the compressor divided to the rate of change at the output of the compressor, in decibels. For instance, a 10 dB change at the input of a 2:1 compressor translates into a 5 dB change at the output. This is illustrated in Fig. 3, a plot of the compressor output in decibels (dB) versus the compressor input, also in dB. The mathematical definition of the compression ration is:

$$C_r = \frac{\partial dB(V_{in}(t))}{\partial dB(V_{out}(t))} \simeq \frac{\Delta dB(V_{in}(t))}{\Delta dB(V_{out}(t))} \quad (33)$$

The infinitesimal difference and the finite difference are equal if the compression ratio is not a function of the input voltage, e.g., 2:1, 1:3. In some audio applications the compressor is on above a threshold. The threshold can be "hard knee" or "soft knee" as shown in Fig. 3. In the case of a "soft knee" compressor, the last two terms of equation (33) are not equal and only the infinitesimal difference should be used.

3.1 Log-Domain Compressor

3.1.1 Feedforward

Let's assume that the input voltage has the following expression:

$$V_{in}(t) = V_p \cdot f(t) \quad (34)$$

where V_p is the peak level and f(t) is the time dependent part of the input signal.

The transfer function of the feedforward compressor, equation (21), can be simplified as follows:

$$\frac{V_{out}(t)}{V_{in}(t)} = \left[\exp(-\tfrac{t}{\tau}) \cdot \left(\int \tfrac{1}{\tau} \cdot \frac{V_{in}^2(t)}{V_r^2} \cdot \exp(\tfrac{t}{\tau}) \cdot dt\right) + c \cdot \exp(-\tfrac{t}{\tau})\right]^{-\frac{k}{2}} \quad (35)$$

Notice that constant c has a contribution only to transient response. If the time is greater than three time constants, the contribution of the second summation term in parenthesis is minimal. Therefore, for $t \Rightarrow \infty$, the output voltage is calculated as follows:

$$V_{out}(t) = V_{in}(t) \cdot \left[\exp(-\tfrac{t}{\tau}) \cdot \left(\int \tfrac{1}{\tau} \cdot \tfrac{V_{in}^2(t)}{V_r^2} \cdot \exp(\tfrac{t}{\tau}) \cdot dt\right)\right]^{-\frac{k}{2}} \quad (36)$$

Substituting (34) in the above equation of the feedforward compressor, equation (36) becomes:

$$V_{out}(t) = V_{in}^{1-k}(t) \cdot \left[\frac{1}{\tau} \cdot \frac{\exp\left(-\frac{t}{\tau}\right)\cdot\left(\int f^2(t)\cdot\exp(\frac{t}{\tau})\cdot dt\right)}{V_r^2 \cdot f^2(t)}\right]^{-\frac{k}{2}} \quad (37)$$

The output voltage in decibels is:

$$dB(V_{out}(t)) = (1-k)\cdot dB(V_{in}(t)) + dB\left(\left[\frac{\int \frac{1}{\tau}\cdot\frac{f^2(t)}{V_r^2}\cdot\exp(\frac{t}{\tau})dt}{f^2(t)\cdot\exp\left(\frac{t}{\tau}\right)}\right]^{-\frac{k}{2}}\right) \quad (38)$$

The compression ratio for a feedforward compressor can be calculated from the above equation as follows:

$$C_r = \frac{\partial dB(V_{in}(t))}{\partial dB(V_{out}(t))} = \frac{1}{1-k} \quad (39)$$

or

$$k = 1 - \frac{1}{C_r} \quad (40)$$

The following table summarizes the compressor functionality as a function of gain factor k.

Table 1

Gain factor k	Compression ratio	Function
$1 < k < \infty$	$-\infty < Cr < 0$	Compression ratio is negative: output decreases as the input increases. The compressor sounds like playing a tape backwards.
$k = 1$	$Cr = \infty$	Infinite compression: output is set to reference level independent of input level. Perfect automatic gain control.
$0 < k < 1$	$1 < Cr < \infty$	Compressor
$k = 0$	$Cr = 1$	No compression
$-\infty < k < 0$	$0 < Cr < 1$	Expander: output change is greater than the input change

3.1.2 Feedback

The transfer function of the feedback compressor, equation (32) can be simplified as follows:

$$\frac{V_{out}(t)}{V_{in}(t)} = \left[\frac{\int \frac{(k+1)}{\tau} \cdot \frac{V_{in}^2(t)}{V_r^2} \cdot \exp\left(\frac{(k+1)\cdot t}{\tau}\right) \cdot dt}{\exp\left(\frac{(k+1)\cdot t}{\tau}\right)} + c \cdot \exp\left(-\frac{(k+1)\cdot t}{\tau}\right) \right]^{-\frac{k}{2\cdot(k+1)}} \qquad (41)$$

As in the case of the feedforward compressor, constant c has a contribution only during transient response. Notice that the RMS detector time constant is divided by a factor of k + 1. If time is greater than three times the equivalent time constant τ, the contribution of the second summation term in parenthesis is minimal. Therefore, for t ⇨ ∝ , the output voltage is calculated as follows:

$$V_{out}(t) = V_{in}(t) \cdot \left[\frac{\int \frac{(k+1)}{\tau} \cdot \frac{V_{in}^2(t)}{V_r^2} \cdot \exp\left(\frac{(k+1)\cdot t}{\tau}\right) \cdot dt}{\exp\left(\frac{(k+1)\cdot t}{\tau}\right)} \right]^{-\frac{k}{2\cdot(k+1)}} \qquad (42)$$

Substituting the definition of the input signal, equation (34), in the above equation, the output voltage becomes:

$$V_{out}(t) = V_{in}^{\frac{1}{k+1}}(t) \cdot \left[\frac{\int \frac{(k+1)}{\tau} \cdot \frac{f^2(t)}{V_r^2} \cdot \exp\left(\frac{(k+1) \cdot t}{\tau}\right) \cdot dt}{f^2(t) \cdot \exp\left(\frac{(k+1) \cdot t}{\tau}\right)} \right]^{-\frac{k}{2 \cdot (k+1)}} \tag{43}$$

The output voltage is expressed in decibels as follows:

$$dB(V_{out}(t)) = \frac{1}{k+1} \cdot dB(V_{in}(t)) + \\ + dB\left(\left[\frac{\int \frac{(k+1)}{\tau} \cdot \frac{f^2(t)}{V_r^2} \cdot \exp\left(\frac{(k+1) \cdot t}{\tau}\right) \cdot dt}{f^2(t) \cdot \exp\left(\frac{(k+1) \cdot t}{\tau}\right)} \right]^{-\frac{k}{2 \cdot (k+1)}} \right) \tag{44}$$

The compression ratio for a feedback compressor can be calculated from the above equation as follows:

$$C_r = \frac{\partial dB(V_{in}(t))}{\partial dB(V_{out}(t))} = k + 1 \tag{45}$$

or

$$k = C_r - 1 \tag{46}$$

The following table summarizes the compressor functionality as a function of gain factor k.

Table 2

Gain factor k	Compression ratio	Function
$0 < k < \infty$	$1 < Cr < \infty$	Compressor: infinite compression is not possible because it requires infinite gain
$k = 0$	$Cr = 1$	No compression
$-1 < k < 0$	$0 < Cr < 1$	Expander: output change is greater than the input change
$k = -1$	$Cr = 0$	Infinite expander: unstable setting
$-\infty < k < -1$	$-\infty < Cr < 0$	Compression ratio is negative: output decreases as the input increases. The compressor sounds like playing a tape backwards.

3.2 Linear-Domain Compression Ratio

In log-domain compressor, the compression ratio is determined by the multiplier factor k. The question is, how can the compression ratio be controlled in the context of an linear-domain compressor ?

Let's define the following notations:

V_{rmsLL} - output of the RMS detector in a linear-domain compressor

V_{rmsLE} - output of the RMS detector in a log-domain compressor

3.2.1 Feedforward

In the previous section it was shown that the transfer function of the compressor based on linear detector and linear VCA matches the transfer function of the compressor based on logarithmic detector and exponential VCA, for k = 1. If both transfer functions match for any value k, then the gain control function, f(Gc), of the linearly-controlled VCA has a different form. Let's equate both transfer functions (4) and (21) :

$$f_1(V_{rmsLL}(t)) = \left[\sqrt{\int \frac{1}{\tau} \cdot \frac{V_{in}^2(t)}{V_r^2} \cdot \exp(\frac{t}{\tau}) \cdot dt + c}\,\right]^{-\frac{k}{2}} \cdot \exp\left(\frac{k \cdot t}{2 \cdot \tau}\right) \quad (47)$$

where f_l is the new gain control function of the linear VCA.

Substituting the linear RMS detector output (3) into equation (47), function f_l has the following form:

$$f_1(V_{rmsLL}(t)) = \left(\frac{1}{V_{rmsLL}}\right)^k \quad (48)$$

Thus, the compression ratio transformation, from the log-domain feedforward compressor to the linear-domain feedforward compressor, is implemented by raising the VCA control voltage to the power of k.

3.2.2 Feedback

It is necessary to check if raising the linear VCA control port to the power k is a solution for the feedback configuration, also. Unfortunately, the transfer functions of the linear-domain and the log-domain feedback compressors can not be equated like in case of the feedforward compressor. The problem is that the factor k is associated with the log filter time constant as shown in equation (32). The solution is to solve the transfer function of the linear-domain feedback compressor using f1(Gc), given by (48), instead of f(Gc) as defined in (2). So, the output voltage is calculated from equation (4) and (48) as follows:

$$V_{out}(t) = \frac{V_{in}(t)}{V_{rms}^k(t)} \quad (49)$$

Substituting equation (49) into equation (6) and applying the property of the derivative function, as shown in (9), the differential equation of the feedback compressor becomes:

$$\frac{\partial V_{rms}^{2\cdot(k+1)}(t)}{\partial t} + \frac{k+1}{\tau} \cdot V_{rms}^{2\cdot(k+1)}(t) = \frac{V_{in}^2(t)}{V_r^2} \cdot \frac{k+1}{\tau} \qquad (50)$$

The above equation is solved the same way as equation (10). The solution is:

$$V_{rms}(t) = \left[\int \frac{k+1}{\tau} \cdot \frac{V_{in}^2(t)}{V_r^2} \cdot \exp\left(\frac{(k+1)\cdot t}{\tau}\right) \cdot dt + c\right]^{\frac{1}{2\cdot(k+1)}} \cdot \exp\left(-\frac{t}{2\cdot\tau}\right) \qquad (51)$$

Substituting the RMS detector output level, equation (51), into equation (49) the transfer function is calculated as follows:

$$\frac{V_{out}(t)}{V_{in}(t)} = \left[\int \frac{k+1}{\tau} \cdot \frac{V_{in}^2(t)}{V_r^2} \cdot \exp\left(\frac{(k+1)\cdot t}{\tau}\right) \cdot dt + c\right]^{-\frac{k}{2\cdot(k+1)}} \cdot \exp\left(\frac{k\cdot t}{2\cdot\tau}\right) \qquad (52)$$

Equation (52) matches the transfer function of the log-domain feedback compressor, equation (32).

The compression ratio transformation implemented by raising the VCA control voltage to the power of k, works for both feedforward and feedback linear-domain compressors.

4 Performance

So far all the results are based on a generic time varying input signal. It was demonstrated that both compressor topologies are identical. Thus, the performance analysis is independent of the RMS detector or VCA. In order to quantify the performance of each compressor topology, the input signal needs to be defined more specific, e.g., sine, square, etc. Let's specify the input voltage as:

$$V_{in}(t) = V_A \cdot \cos(\omega t) \qquad (53)$$

where V_A is the peak level and ω is the signal frequency in radians per second.

4.1 Feedforward

Let's substitute the definition of the input signal (53) into the transfer function of a feedforward compressor, equation (21):

$$\frac{V_{out}(t)}{V_{in}(t)} = \left[\frac{V_A^2}{2\cdot\tau\cdot V_r^2} \cdot E(t) + c\right]^{-\frac{k}{2}} \cdot \exp\left(\frac{k\cdot t}{2\cdot\tau}\right) \qquad (54)$$

where:

$$E(t) = \int \exp\left(\frac{t}{\tau}\right) \cdot dt + \int \cos(2\omega t) \cdot \exp\left(\frac{t}{\tau}\right) \cdot dt \qquad (55)$$

and the following trigonometric property is used:

$$\cos^2(\omega t) = \frac{1+\cos(2\omega t)}{2} \tag{56}$$

The first integral in E(t) is solved as follows:

$$\int \exp(\tfrac{t}{\tau}) \cdot dt = \tau \cdot \exp(\tfrac{t}{\tau}) \tag{57}$$

The second integral in E(t) is solved as follows:

$$\begin{aligned} &\int \cos(2\omega t) \cdot \exp(\tfrac{t}{\tau}) \cdot dt = \tau \cdot \exp(\tfrac{t}{\tau}) \cdot \cos(2\omega t) + \\ &+ 2\omega\tau \cdot \int \exp(\tfrac{t}{\tau}) \cdot \sin(2\omega t) \cdot dt = \tau \cdot \exp(\tfrac{t}{\tau}) \cdot \cos(2\omega t) + \\ &+ 2\omega\tau \cdot \left[\tau \cdot \exp(\tfrac{t}{\tau}) \cdot \sin(2\omega t) - 2\omega\tau \cdot \int \cos(2\omega t) \cdot \exp(\tfrac{t}{\tau}) \cdot dt\right] \end{aligned} \tag{58}$$

Notice that the last term of equation (58) is the original integral. So, the above equation can be solved for the second integral term of E(t) as follows:

$$\int \cos(2\omega t) \cdot \exp(\tfrac{t}{\tau}) \cdot dt = \tau \cdot \exp(\tfrac{t}{\tau}) \cdot \frac{\cos(2\omega t)+2\omega\tau\cdot\sin(2\omega t)}{1+(2\omega\tau)^2} \tag{59}$$

Substituting the integral results (57) and (59) into equation (55), E(t) is evaluated as follows:

$$E(t) = \tau \cdot \exp(\tfrac{t}{\tau}) \cdot G(\omega t, \tau) \tag{60}$$

where $G(\omega t, \tau)$ is defined as follows:

$$G(\omega t, \tau) = 1 + \frac{\cos(2\omega t)+2\omega\tau\cdot\sin(2\omega t)}{1+(2\omega\tau)^2} \tag{61}$$

Finally, substituting equations (40) and (60) into (54) the transfer function of the feedforward compressor is expressed as a function of input signal and compression ratio as follows:

$$\frac{V_{out}(t)}{V_{in}(t)} = \left[\frac{\left(\frac{V_A}{\sqrt{2}}\right)^2}{V_r^2} \cdot G(\omega t, \tau) + c \cdot \exp(-\tfrac{t}{\tau})\right]^{\frac{1-C_r}{2\cdot C_r}} \tag{62}$$

The transfer function of a compressor is basically its gain function. The transfer function in equation (62) has a few terms, each with its own significance. The first term in parenthesis is the steady state solution: ripple and dc. The ripple is given by $G(\omega t,\tau)$ as shown in (61). The ripple is the residual ac left after the first order low pass filter in the RMS detector. It modulates the compressor gain and it is a major source of distortion. The dc term is the compressor ideal gain if the RMS detector low pass filter would have been ideal. The second summation term in the parenthesis decays with time and it has a significant contribution only during transient response.

4.1.1 Ideal gain transfer function

The ideal gain transfer function is the steady state solution of transfer function (62) in the presence of an RMS detector with ideal integrator, or no ripple. In this case, the transfer function (62) can be written as:

$$\frac{V_{out}(t)}{V_{in}(t)} = \left[\frac{\left(\frac{V_A}{\sqrt{2}}\right)^2}{V_r^2}\right]^{\frac{1-C_r}{2\cdot C_r}} \tag{63}$$

The transfer function was rearranged to show that the compressor gain is proportional to the RMS value of the input signal. Indeed, the RMS value of a sine is the its peak value divided by the square root of two. For commonly used time constants, in the range of tens of milliseconds, this formula is quite safe to use when calculating the compressor gain. In this case, the error caused by ripple is small. For instance, for a time constant of 35 ms, the peak value of $G(\omega t,\tau)$ at 100 Hz is about 1.02 .

4.1.2 Ripple and Harmonic Distortion

As mentioned before, the ripple modulates the compressor gain causing distortion. Substituting equations (53) and (61) into equation (62), the output voltage steady state solution is calculated as follows:

$$V_{out}(t) = V_A \cdot \cos(\omega t)\left[\frac{\left(\frac{V_A}{\sqrt{2}}\right)^2}{V_r^2}\cdot\left(1+\cos(2\omega t-\varphi)\cdot\cos\varphi\right)\right]^{\frac{1-C_r}{2\cdot C_r}} \tag{64}$$

where:

$$\tan\varphi = 2\omega\tau \tag{65}$$

The Taylor series of a binomial is:

$$(1+x)^n = 1 + n\cdot x + \frac{n\cdot(n-1)}{2!}\cdot x^2 + \frac{n\cdot(n-1)\cdot(n-2)}{3!}\cdot x^3 + \ldots \tag{66}$$

If x is small, a good approximation is to consider only first two terms of the Taylor series. In this case, this can be done if $\cos(\varphi)$ is small. The greatest value of $\cos(\varphi)$ is at low frequencies and/or small time constants. For commonly used time constants this approximation can be made. For instance, for a time constant of 35 ms, $\cos(\varphi)$ at 20 Hz is about 0.1 and less for higher frequencies . Thus, taking into account Taylor series (66), the output voltage can be approximated as follows:

$$V_{out}(t) = V_A \cdot \left(\frac{\frac{V_A}{\sqrt{2}}}{V_r} \right)^{\frac{1-C_r}{C_r}} \cdot \cos(\omega t) \cdot \left(1 + \frac{1-C_r}{2 \cdot C_r} \cdot \cos\varphi \cdot \cos(2\omega t - \varphi)\right)$$

(67)

Recall from trigonometry the following property of cosine function:

$$\cos a \cdot \cos b = \frac{\cos(a+b)+\cos(a-b)}{2} \tag{68}$$

The above trigonometric property is applied to equation (67) as follows:

$$V_{out}(t) = V_A \cdot \left(\frac{\frac{V_A}{\sqrt{2}}}{V_r} \right)^{\frac{1-C_r}{C_r}} \cdot$$

$$\cdot \left(\cos(\omega t) + \frac{1-C_r}{4 \cdot C_r} \cdot \cos\varphi \cdot (\cos(3\omega t - \varphi) + \cos(\omega t - \varphi))\right)$$

(69)

The spectra of the output voltage contains the fundamental and the third harmonic of the fundamental. The total harmonic distortion is the square root of the sum of the square of all harmonics divided to the square of the fundamental. If all the harmonics are normalized to the amplitude of the fundamental, then the total harmonic distortion is the square root of the sum of the square of all harmonics. In this case, only the third harmonic exists and the total harmonic distortion is equal to the normalized third harmonic amplitude. This is because only the first term of the Taylor series is considered. If more terms are taken into account then the spectra of the output signal is a sum of odd harmonics. This can be demonstrated by noticing that the input signal, $\cos(\omega t)$, multiplies a $\cos^n(2\omega t)$ term (see equation 64). This can be written as follows:

$$\cos(\omega t) \cdot \cos^n(2\omega t) = \cos(\omega t) \cdot \sum_m a_m \cdot \cos(m \cdot 2\omega t) =$$

$$= \sum_m a_m \cdot \cos(\omega t) \cdot \cos(m \cdot 2\omega t) =$$

$$= \sum_m \frac{a_m}{2} \cdot (\cos((2m+1) \cdot \omega t) + \cos((2m-1) \cdot \omega t))$$

(70)

where a_m are a series of real coefficients.

Thus, the result of the multiplication is a sum of odd harmonics. Notice another trigonometric property of the cosine function:

$$\cos(\omega t) - b \cdot \cos(\omega t - \varphi) = \sqrt{1 + b^2 - 2 \cdot b \cdot \cos\varphi} \cdot \cos(\omega t + \varphi_1) \tag{71}$$

where φ_1 is defined as:

$$\tan\varphi_1 = \frac{b \cdot \sin\varphi}{1 - b \cdot \cos\varphi} \tag{72}$$

Applying the cosine property (71) into equation (69) and normalizing the spectra, the frequency content of the output voltage is:

$$V_{out}(\omega t) = \cos(\omega t + \varphi_1) - \frac{\frac{C_r - 1}{4 \cdot C_r}}{\sqrt{\left(\frac{3 \cdot C_r + 1}{4 \cdot C_r}\right)^2 + (2\omega\tau)^2}} \cdot \cos(3\omega t - \varphi) \tag{73}$$

where φ_1 is defined as:

$$\tan\varphi_1 = \frac{\frac{C_r - 1}{4 \cdot C_r} \cdot (2\omega\tau)}{\frac{3 \cdot C_r + 1}{4 \cdot C_r} + (2\omega\tau)^2} \tag{74}$$

The distortion of the feedforward compressor is equal to the coefficient of the third harmonic as follows:

$$THD_{FF} = \frac{\left|\frac{C_r - 1}{4 \cdot C_r}\right|}{\sqrt{\left(\frac{3 \cdot C_r + 1}{4 \cdot C_r}\right)^2 + (2\omega\tau)^2}} \tag{75}$$

Equation (75) is plotted in Fig. 4, solid curve, for 20:1 compression ratio. It's interesting to notice that the distortion at low frequencies is flat. At high frequencies, distortion decreases with frequency at a rate of 6 dB/octave. In a real compressor, the distortion meter measures THD+N, or total harmonic distortion plus noise. Therefore, when the harmonic distortion is lower than the noise floor, the distortion curve is flat also, this time due to the noise of the compressor. Since the distortion is flat and then decreases at a rate of 6 dB/octave it has a "-3dB" point at:

$$\omega_{FF_{-3dB}} = \frac{1}{2 \cdot \tau} \cdot \frac{3 \cdot C_r + 1}{4 \cdot C_r} \tag{76}$$

For a 35 ms time constant, this point moves from 1.7 Hz at infinite compression, to 2 Hz at 2:1 compression and to 2.3 Hz at no compression. So, the "-3dB" corner frequency is around 2 Hz. Below the corner frequency, the distortion predicted by formula (75) is 33% at infinite compression and 14% at 2:1 compression. At 1 kHz, the distortion drops to 0.07% at infinite compression and about 0.03% at 2:1 compression.

Figure 5 shows the FFT of a 500 Hz output signal given by equation (64), for an RMS detector time constant τ of 350 μs and infinite compression. The input signal is a 500 Hz cosine. Notice that only the third harmonic is in the output signal spectra. The distortion calculated from FFT is 11 %. The same distortion calculated by the formula (75) is 10.7 % . Lowering the RMS detector time constant to τ = 100 μs, more odd terms show up in the FFT, as shown in figure 6. The distortion calculated from the FFT is 31.7 % and since formula (75) takes into account only the third harmonic it predicts 25.6 %. Notice that no even harmonics are present in the output signal spectra, as demonstrated in equation (70).

4.1.3 Transient response

Transient behavior occurs when the input signal changes from one level to another. Let's define the following input signals around t = 0:

$$\begin{aligned} t_{0-} &: V_{in}(t) = V_A \cdot \cos(\omega t) \\ t_{0+} &: V_{in}(t) = V_B \cdot \cos(\omega t) \end{aligned} \tag{77}$$

where t_{0-} is the time just before t = 0 and t_{0+} is the time just after t = 0.

The compressor gain just before t = 0 is the steady state solution of the transfer function for the input level at t_{0-}:

$$\frac{V_{out}(t)}{V_{in}(t)} = \left[\frac{\left(\frac{V_A}{\sqrt{2}}\right)^2}{V_r^2} \cdot G(\omega t, \tau)\right]^{\frac{1-C_r}{2 \cdot C_r}} \tag{78}$$

When the input signal changes at t = 0, the compressor takes a finite time to react and its gain is still the one at t_{0-} . Thus, the gain calculated in equation (78) is equated with the transfer function of the feedforward compressor, equation (62), for the input signal as defined for t_{0+} :

$$\left[\frac{\left(\frac{V_A}{\sqrt{2}}\right)^2}{V_r^2} \cdot G(\omega 0, \tau)\right]^{\frac{1-C_r}{2 \cdot C_r}} = \left[\frac{\left(\frac{V_B}{\sqrt{2}}\right)^2}{V_r^2} \cdot G(\omega 0, \tau) + c\right]^{\frac{1-C_r}{2 \cdot C_r}} \tag{79}$$

Equation (79) is solved for constant c as follows:

$$c = \frac{G(\omega 0, \tau)}{V_r^2} \cdot \left(\left(\frac{V_A}{\sqrt{2}}\right)^2 - \left(\frac{V_B}{\sqrt{2}}\right)^2\right) = \frac{1}{2} \cdot \frac{G(\omega 0, \tau)}{V_r^2} \cdot \left(V_A^2 - V_B^2\right) \tag{80}$$

Constant c is proportional to the difference between the RMS values of the signal before t = 0 and the signal after t = 0. Factor G(ω0, τ) is added to match the phase of the ripple before and after t = 0. Substituting constant c in the transfer function of the feedforward compressor, equation (62), and ignoring the effect of the ripple, the transient transfer functions becomes:

$$\frac{V_{out}(t)}{V_{in}(t)} = \left[\frac{1}{2} \cdot \frac{V_B^2}{V_r^2} + \frac{1}{2} \cdot \frac{(V_A^2 - V_B^2)}{V_r^2} \cdot \exp(-\tfrac{t}{\tau})\right]^{\frac{1-C_r}{2 \cdot C_r}} \tag{81}$$

The above transfer function is the same for attack and release time. A plot of attack and release transfer function is shown in Fig. 7 . The shape of the attack and release envelope is the shape of a decaying and raising exponential function with a time constant τ .

4.2 Feedback

The transfer function of the feedback compressor is determined in the same manner as the feedforward compressor. The input signal definition (53) is substituted into equation (32) as follows:

$$\frac{V_{out}(t)}{V_{in}(t)} = \left[\frac{(k+1) \cdot \left(\frac{R2}{R1}\right)^2 \cdot V_A^2}{2 \cdot \tau \cdot V_r^2} \cdot D(t) + c\right]^{-\frac{k}{2 \cdot (k+1)}} \cdot \exp\left(\frac{k \cdot t}{2 \cdot \tau}\right) \tag{82}$$

where D(t) has the following form:

$$D(t) = \int \exp(\tfrac{t}{\tau} \cdot (k+1)) \cdot dt + \int \cos(2\omega t) \cdot \exp(\tfrac{t}{\tau} \cdot (k+1)) \cdot dt \tag{83}$$

Solving the integrals in equation (83), D(t) is written as follows:

$$D(t) = \frac{\tau}{(k+1)} \cdot \exp(\tfrac{t}{\tau} \cdot (k+1)) \cdot H(\omega t, \tau) \tag{84}$$

where $H(\omega t, \tau)$ is defined as follows:

$$H(\omega t, \tau) = 1 + \frac{\cos(2\omega t) + 2\omega \frac{\tau}{k+1} \cdot \sin(2\omega t)}{1 + \left(2\omega \frac{\tau}{k+1}\right)^2} \tag{85}$$

Substituting equation (46) and equation (84) into equation (82), the transfer function of the feedback compressor can be expressed as a function of the input signal and compression ratio as follows:

$$\frac{V_{out}(t)}{V_{in}(t)} = \left[\frac{\left(\frac{V_A}{\sqrt{2}}\right)^2}{V_r^2} \cdot H(\omega t, \tau) + c \cdot \exp\left(-\frac{t}{\left(\frac{\tau}{C_r}\right)}\right)\right]^{\frac{1-C_r}{2 \cdot C_r}} \tag{86}$$

The same notes that applied to the feedforward compressor are correct for the feedback configuration. The major difference is that the time constant is divided by the compression ratio.

4.2.1 Ideal transfer function

The ideal transfer function of the feedback compressor is the steady state solution of the transfer function (86) with an ideal low pass filter in the RMS detector, or no ripple:

$$\frac{V_{out}(t)}{V_{in}(t)} = \left[\frac{\left(\frac{V_A}{\sqrt{2}}\right)^2}{V_r^2}\right]^{\frac{1-C_r}{2 \cdot C_r}} \tag{87}$$

The feedback compressor gain is proportional to the RMS value of the input signal. If the equivalent time constant, that is the RMS detector time constant divided by the compression ratio, is the range of tens of milliseconds, the above formula can be safely used to calculated the gain of the feedback compressor.

4.2.2 Ripple and Harmonic Distortion

As in the case of the feedforward compressor, the ripple factor H(ωt, τ) modulates the feedback compressor gain and causes distortion. Substituting the definition of the input signal (53) and ripple (85) into the transfer function (86), the output voltage is calculated as follows:

$$V_{out}(t) = V_A \cdot \cos(\omega t)\left[\frac{\left(\frac{V_A}{\sqrt{2}}\right)^2}{V_r^2} \cdot (1 + \cos(2\omega t - \theta) \cdot \cos\theta)\right]^{\frac{1-C_r}{2 \cdot C_r}} \tag{88}$$

where:

$$\tan\theta = 2\omega\frac{\tau}{C_r} \tag{89}$$

Equation (88) can be expanded into a Taylor series using the binomial series (66):

$$V_{out}(t) = V_A \cdot \left(\frac{\frac{V_A}{\sqrt{2}}}{V_r}\right)^{\frac{1-C_r}{C_r}} \cdot \cos(\omega t) \cdot \left(1 + \frac{1-C_r}{2 \cdot C_r} \cdot \cos\theta \cdot \cos(2\omega t - \theta)\right) \tag{90}$$

Using the property of the cosine function (68), the above equation is rewritten as follows:

$$V_{out}(t) = V_A \cdot \left(\frac{\frac{V_A}{\sqrt{2}}}{V_r} \right)^{\frac{1-C_r}{C_r}} \cdot$$

$$\cdot \left(\cos(\omega t) + \tfrac{1-C_r}{4 \cdot C_r} \cdot \cos\theta \cdot (\cos(3\omega t - \theta) + \cos(\omega t - \theta)) \right) \tag{91}$$

The spectra of the output voltage contains the fundamental and the third harmonic. The more terms of Taylor series are considered, the more odd harmonics are added to the sum. For commonly used time constants, in the tens of milliseconds range, the third harmonic is a good approximation. Applying the cosine property (71) to equation (91) and normalizing the spectra to the fundamental, the frequency content of the output voltage follows:

$$V_{out}(\omega t) = \cos(\omega t + \theta_1) - \frac{\frac{C_r - 1}{4 \cdot C_r}}{\sqrt{\left(\frac{3 \cdot C_r + 1}{4 \cdot C_r}\right)^2 + \left(2\omega \frac{\tau}{C_r}\right)^2}} \cdot \cos(3\omega t - \theta) \tag{92}$$

where θ_1 is defined as:

$$\tan\theta_1 = \frac{\frac{C_r - 1}{4 \cdot C_r} \cdot \left(2\omega \frac{\tau}{C_r}\right)}{\frac{3 \cdot C_r + 1}{4 \cdot C_r} + \left(2\omega \frac{\tau}{C_r}\right)^2} \tag{93}$$

The feedback compressor, as well as the feedforward compressor, has a built-in inherent phase shift or delay, θ_1. The delay is zero at dc, or if there is no compression, $C_r = 1$, or if the time constant is zero. The distortion of the feedback compressor is equal to the coefficient of the third harmonic as follows:

$$THD_{FB} = \frac{\left| \frac{C_r - 1}{4 \cdot C_r} \right|}{\sqrt{\left(\frac{3 \cdot C_r + 1}{4 \cdot C_r}\right)^2 + \left(2\omega \frac{\tau}{C_r}\right)^2}} \tag{94}$$

Equation (94) is plotted in Fig. 4, dashed curve, for 20:1 compression ratio. The distortion at low frequencies is also flat. At high frequencies, distortion decreases with frequency at a rate of 6 dB/octave. The distortion "-3dB" corner frequency for the feedback compressor is:

$$\omega_{FB_{-3dB}} = \frac{C_r}{2 \cdot \tau} \cdot \frac{3 \cdot C_r + 1}{4 \cdot C_r} \tag{95}$$

For similar RMS detector time constant, the feedback compressor "-3dB" frequency corner is multiplied by the compression ratio. This is especially important at high compression ratios and because the higher distortion plateau is extended to higher frequencies. For instance, for a 35 ms time constant, at a compression ratio of 20:1 the corner frequency for the feedforward compressor is 1.7 Hz compared to 35 Hz for the feedback configuration. However, if the RMS detector time constant is adjusted by multiplying it by the compression ratio, then there is no difference in performance between feedforward and feedback compressor.

4.2.3 Transient response

The input signals for the transient response are defined in (77). The feedback compressor gain right before the input signal transition is:

$$\frac{V_{out}(t)}{V_{in}(t)} = \left[\frac{\left(\frac{V_A}{\sqrt{2}}\right)^2}{V_r^2} \cdot H(\omega t, \tau)\right]^{\frac{1-C_r}{2 \cdot C_r}} \tag{96}$$

When the input signal changes in level at t = 0, the compressor gain requires finite time to change. Therefore, the gain at t_{0+} equals the gain at t_{0-}:

$$\left[\frac{\left(\frac{V_A}{\sqrt{2}}\right)^2}{V_r^2} \cdot H(\omega 0, \tau)\right]^{\frac{1-C_r}{2 \cdot C_r}} = \left[\frac{\left(\frac{V_B}{\sqrt{2}}\right)^2}{V_r^2} \cdot H(\omega 0, \tau) + c\right]^{\frac{1-C_r}{2 \cdot C_r}} \tag{97}$$

The above equation is solved for constant c as follows:

$$c = \frac{H(\omega 0,\tau)}{V_r^2} \cdot \left(\left(\frac{V_A}{\sqrt{2}}\right)^2 - \left(\frac{V_B}{\sqrt{2}}\right)^2\right) = \frac{H(\omega 0,\tau)}{2 \cdot V_r^2} \cdot \left(V_A^2 - V_B^2\right) \tag{98}$$

The ripple value at t = 0, H(ωt, τ), is only necessary to match the phase of ripple at the transition.

Substituting constant c in equation (86) and neglecting the ripple, the transient transfer function has the following form:

$$\frac{V_{out}(t)}{V_{in}(t)} = \frac{R2}{R1} \cdot \left[\frac{V_B^2}{2 \cdot V_r^2} + \frac{\left(V_A^2 - V_B^2\right)}{2 \cdot V_r^2} \cdot \exp\left(-\frac{t}{\left(\frac{\tau}{C_r}\right)}\right)\right]^{\frac{1-C_r}{2 \cdot C_r}} \tag{99}$$

The transient response of the feedback compressor is similar to the feedforward compressor with the difference of the RMS detector time constant is divided by the compressor ratio. The same attack and release curves shown in Fig. 7. can be obtained by setting the RMS detector time constant to τ = 35 ms * Cr. In this case, the compression ratio is 20:1 and the RMS time constant is τ = 700 ms.

The difference between the transient response of the feedforward (solid curve) and feedback (dashed curve) compressors is shown in Fig. 8 . The RMS time constant and compression ratio are equal, $\tau = 35$ ms and Cr = 20, respectively. Notice that the feedback compressor is 20 times faster than the feedforward compressor.

4.3 Other Factors that Influence Performance

There are second order effects that could influence the compressor performance. For instance, it was shown that the total harmonic distortion of a compressor, due to detector ripple, is a sum of odd harmonics, only. In reality, a non-ideal VCA can contribute even harmonics to the spectra. A not quite symmetrical rectifier in the RMS detector adds unwanted harmonics to the compressor spectra, as well. In DSP implementations the ADCs and DACs, add harmonics of their own. Ultimately, the noise of the devices, VCA, RMS detector operational amplifiers and resistors add to the distortion plus noise number and limit the dynamic range of the detector or compressor.

The transient response can be also slightly different than the one predicted by the theory. The timing capacitors have finite equivalent series resistance, ESR, anywhere from fractions of ohm to a few ohms. In order to account for ESR, the equivalent schematics shown in figures x and y, change by adding a resistor in series with the timing capacitor. The mathematical model is very complex and it is not shown in this paper. The result is a faster RMS detector. Another effect of the ESR is ripple increase and eventually higher distortion.

The integrator of the logarithmic output RMS detector may be based on a log filter [7]. The structure of the log filter is similar to the linear R-C model but the resistor is replaced by a diode. The diode exhibits dynamic conductance that is a function of the current flowing through it. More current translates into increased conductance. A resistor in series with the diode limits the maximum conductance and substantially complicates the mathematical model of the RMS detector. The result is an RMS detector with slower attack time. The series resistor is a "natural" component since all diodes have finite series resistor. Also, improper layout design of an RMS detector IC metal mask could add unnecessary series resistor.

A linear based compressor can suffer from limitation such as current and voltage availability and ultimately power dissipation. If the filter in the linear RMS detector is made of a 3.5 kΩ resistor and 10 μF capacitor (35 ms time constant), the peak current could be as high as 86 mA when 303 V is applied to the resistor (corresponds to peak +24 dBu). This translates to 24 W peak power required from the power supply.

The logarithmic RMS detector can suffer from current limitations as well. The current flowing through the logarithmic filter diode is proportional to the square of the input current [7]. A change of +20 dB in the input voltage can translate into a +40 dB current jump in the log filter diode. If the reference level current is set to 8.5 uA and the input resistor is 10 kΩ [10], the maximum peak current for +24 dBu input signal is 3 mA. A proper RMS detector design can provide that much current. However, if the logarithmic RMS detector input resistor has a lower value, then current clipping can occur. The power dissipation in this case is not an issue, since the maximum voltages are around two diode drops or about 1.3 V.

5 Compressor with Threshold

In many applications the compressor is implemented with threshold. The input signal is compressed above threshold, only. Below threshold the gain of the VCA is one. The threshold circuit is usually part of the k multiplier block as shown in Fig. 1 and Fig. 2 . The transfer function of a compressor with threshold is shown in Fig. 3 . The "soft knee" gives the compressor a nice sonic transition.

In Fig. 9 is plotted the transient response of an above threshold compressor (for clarity, only the signal envelope is shown). The threshold is set to 1 Vpeak, dash-dot line. The RMS detector output is shown in the lower plot of Fig. 9 . Only the signal above the threshold line is applied to the VCA. If the RMS output is below threshold, the VCA gain is one.

Before t = 0, the input signal is below threshold and the VCA gain is one. Between 0 ms and 150 ms the input signal is 6 dB above the threshold. The RMS detector output increases but it does not change the VCA gain until it reaches the threshold. Thus, until the RMS detector output reaches the threshold, the VCA gain is one and the compressor puts out the whole input signal. This is an important difference between the compressor with and without threshold. In an amplifier and/or driver protection application, the duration of time the protected device has full power applied is very important and it can determine the compressor time constant. Between 150 ms and 400 ms, the signal drops from 6 dB to 3 dB above threshold. The compressor output is similar to the one shown in Fig. 8 . At 400 ms, the input signal drops to 6 dB below threshold and the RMS detector output slowly decreases. Although the input signal is below threshold, the VCA gain is modulated by the RMS detector, and the output signal is compressed until the RMS detector output reaches the threshold.

6 Conclusion

At the beginning of the paper, it was demonstrated that the compressor based on logarithmic output RMS detector - exponentially controlled VCA and the one based on linear output RMS detector - linearly controlled VCA are identical. Therefore, all compressor implementations, hardware or software, share similar performance.

The following table summarizes the performance difference between feedforward and feedback compressor topologies.

Table 3

	Feedforward	Feedback	Notes
THD at low frequencies and/or fast time constant	$\frac{C_r-1}{3 \cdot C_r+1}$	$\frac{C_r-1}{3 \cdot C_r+1}$	The distortion is equal at low frequencies.
THD at higher frequencies or slow time constant	$\frac{C_r-1}{8 \cdot C_r \cdot \omega \cdot \tau}$	$\frac{C_r-1}{8 \cdot \omega \cdot \tau}$	The feedback compressor THD is Cr times the feedforward THD.
THD "-3dB" frequency	$\frac{1}{2 \cdot \tau} \cdot \frac{3 \cdot C_r+1}{4 \cdot C_r}$	$\frac{C_r}{2 \cdot \tau} \cdot \frac{3 \cdot C_r+1}{4 \cdot C_r}$	The feedback "-3 dB" point is Cr times the feedforward compressor "-3 dB" point.
Time constant	τ	$\frac{\tau}{C_r}$	For equal RMS detector time constants, the feedback compressor time constant is divided Cr times.

A few conclusions can be drawn from the above table. The performance of the feedback compressor is directly affected by the compression ratio. For similar transient response, the RMS detector time constant in the feedback compressor needs to be multiplied by the compression ratio. This is easy in fixed compression ratio applications. In feedback compressors with adjustable compression ratio the circuit necessary to keep the same time constant is quite complex. The distortion of the feedback compressor is also multiplied by the compression ratio. Thus, a feedforward configuration is more appropriate for a compressor.

Theoretically, infinite compression ratio is impossible in feedback compressors. However, a compression ratio of 20 or more approximates an infinite compressor The major drawback is that the distortion at 1 kHz is 1.4 % compared to only 0.07 % for the feedforward configuration. In order to maintain the same performance, the RMS detector timing capacitor has to be increased 20 times. Thus, the capacitor value can increase to a couple hundred µF. The capacitor is more expensive and requires more PCB area.

In a feedback compressor topology, the dynamic range at the input of the RMS detector is smaller than the dynamic range at the input of the RMS detector in the feedforward compressor topology. The reason is that the output voltage is already compressed. Thus, the feedback compressor is more appropriate in applications that require a large dynamic range that the RMS detector can not handle.

The feedback configuration is suitable for expander applications. The compressor becomes an expander when output level changes are greater than input level changes. In this case the compression ratio is between zero and one. The distortion performance is better than the feedforward expander. This can be explained by the fact that the gain factor k is smaller for feedback configuration compared to feedforward. Therefore, there is less ripple passed from the RMS detector to the VCA and as a consequence less distortion. The timing capacitor is multiplied by 1/Cr factor, also. Thus, smaller timing capacitor is needed. Since RMS detector time constant is divided by the compression, the same disadvantage remains in applications where the compression ratio is a variable.

7 Acknowledgment

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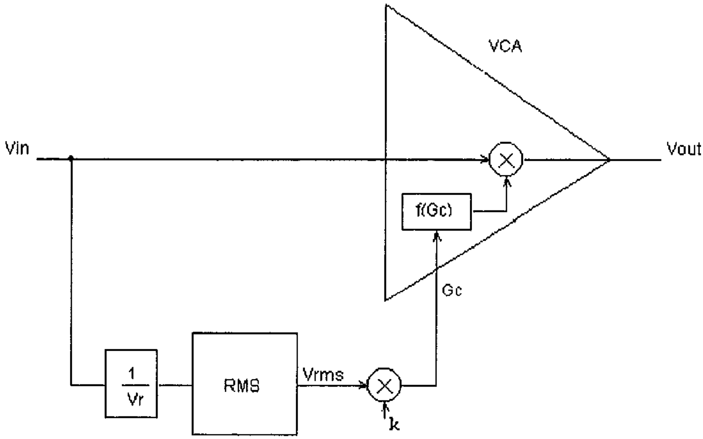

Figure 1. Feedforward compressor topology.

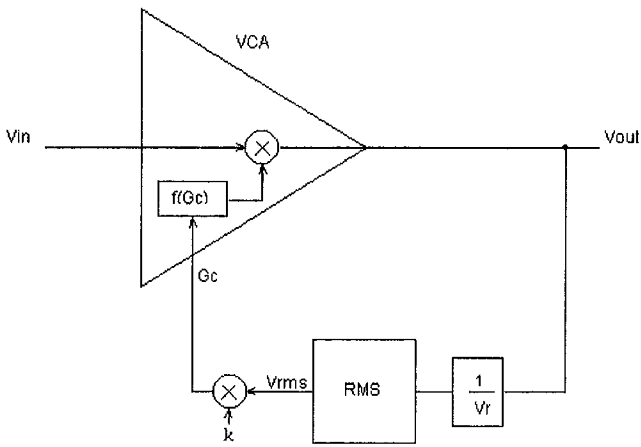

Figure 2. Feedback compressor topology.

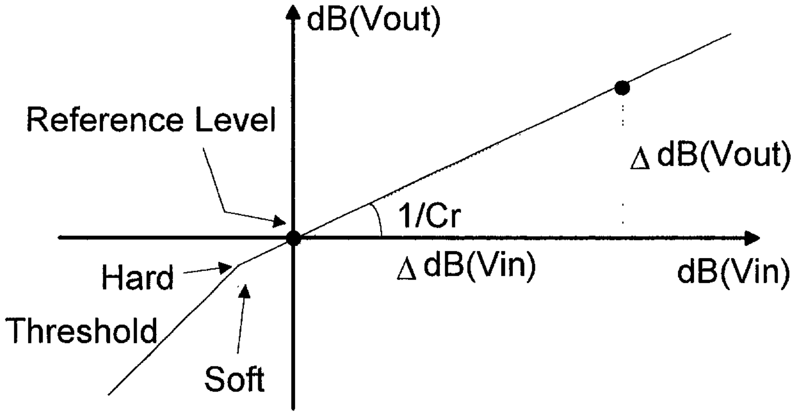

Figure 3. Compressor transfer function.

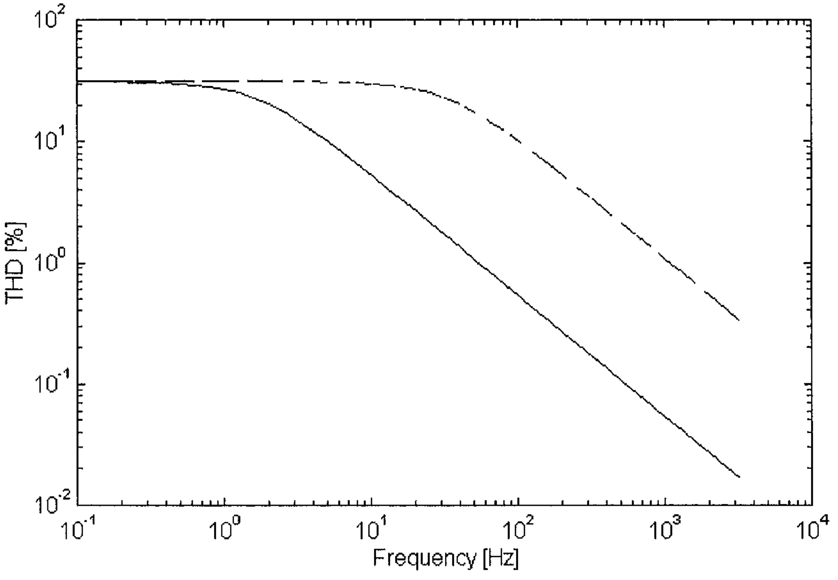

Figure 4. THD versus frequency. Solid curve feedforward compressor and dashed curve feedback compressor. Compression ratio is 20, RMS detector time constant 35 ms.

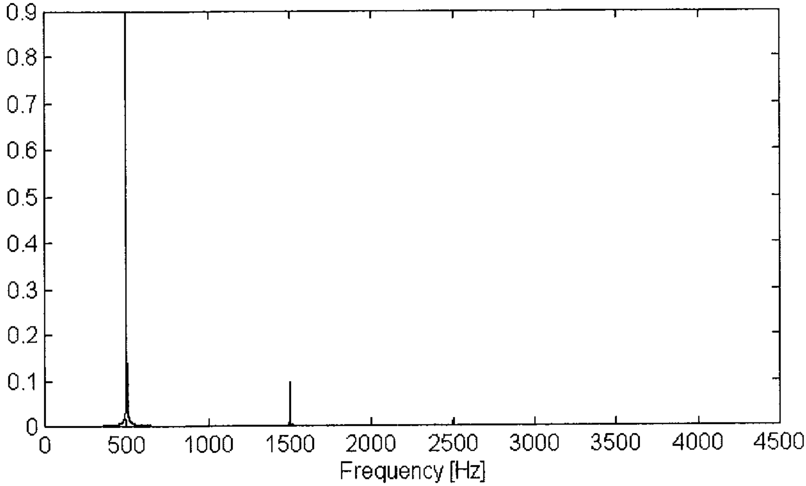

Figure 5. FFT of the feedforward compressor output. RMS detector time constant 350 μs

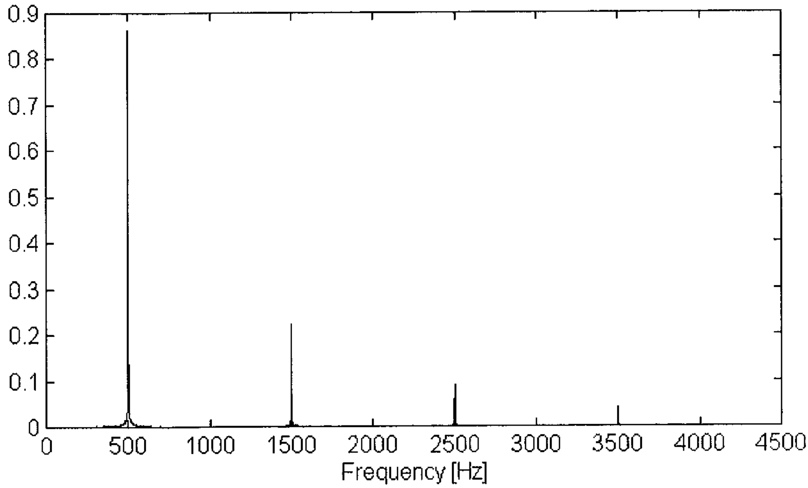

Figure 6. FFT of the feedforward compressor output. RMS detector time constant 100 μs

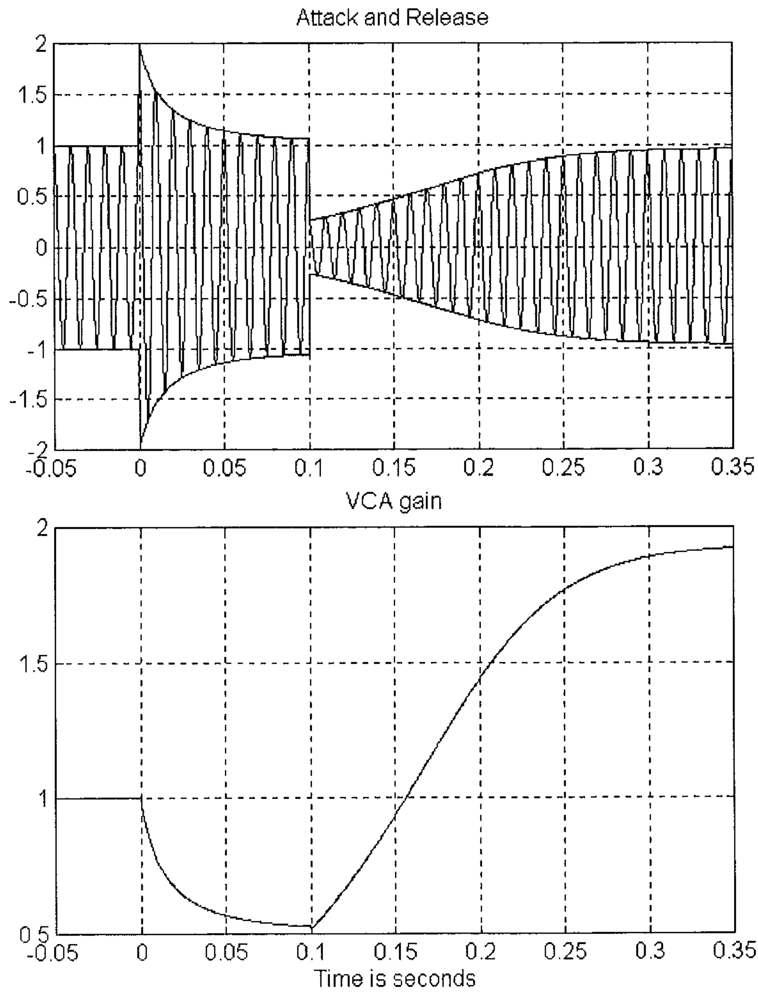

Figure 7. Feedforward compressor: upper plot, attack and release gain modulation, lower plot, VCA gain. RMS detector time constant 35 ms, reference level 1 Vpeak, signal frequency 100 Hz, compression ratio 20:1; signal level: 6 dB above reference level during attack, 6 dB below reference level during release.

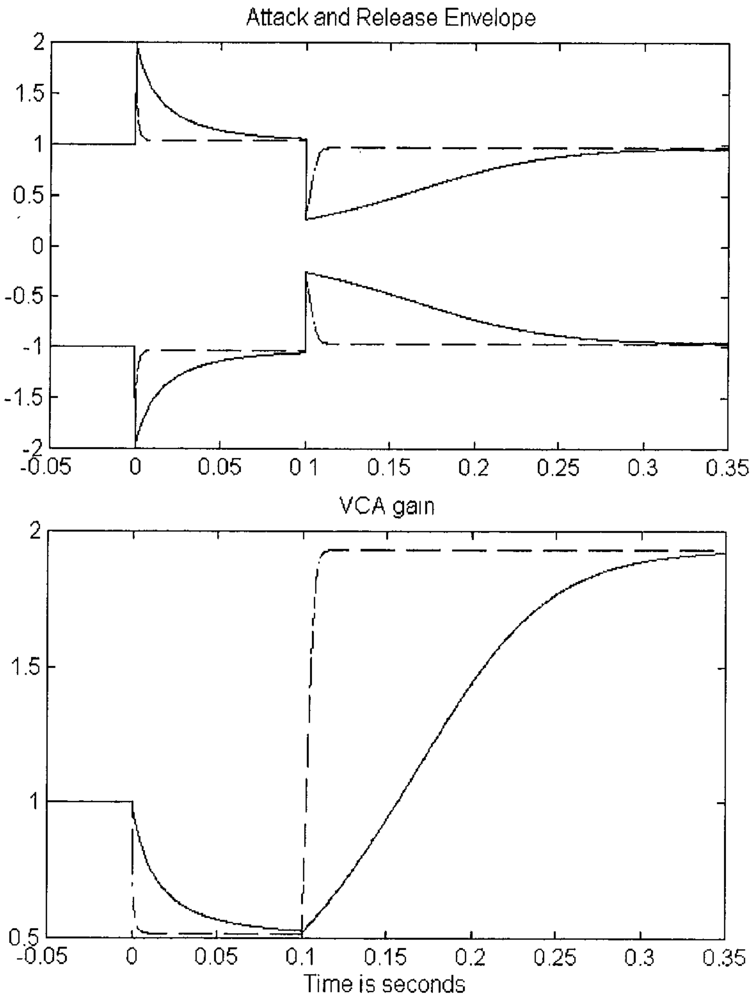

Figure 8. Feedforward, solid curve, and feedback, dashed curve; compressor. Upper plot: attack and release gain modulation. Lower plot: VCA gain. RMS detector time constant 35 ms, reference level 1 Vpeak, signal frequency 100 Hz, compression ratio 20:1. Level: 6 dB above reference level during attack, 6 dB below reference level during release.

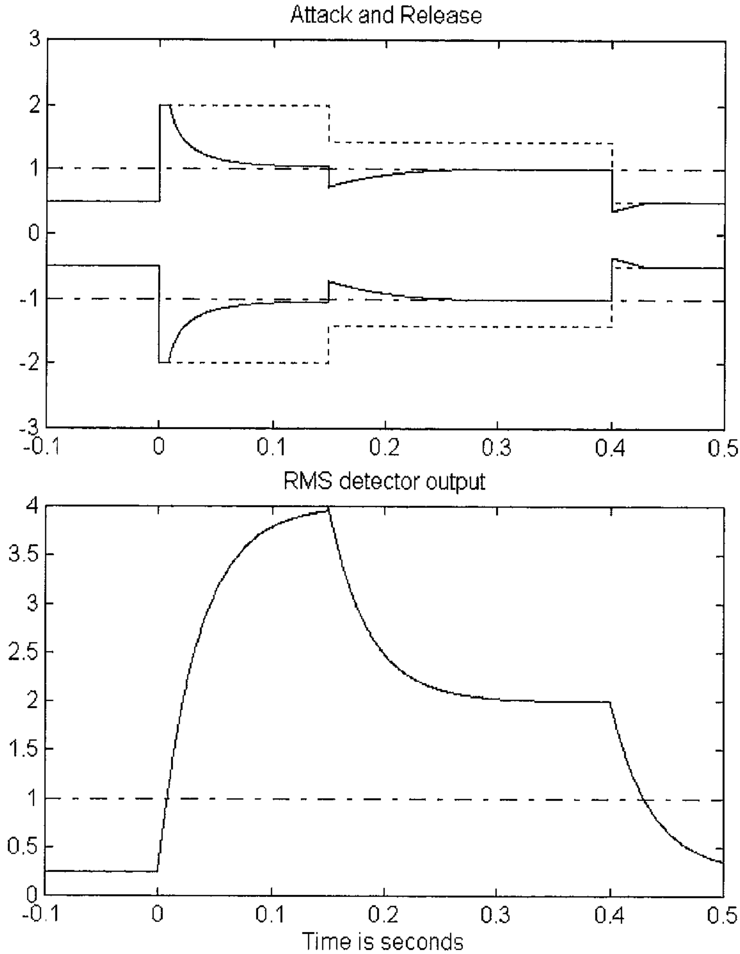

Figure 9. Feedforward compressor with threshold. Upper plot: attack and release gain modulation, solid curve, input signal peak amplitude, dotted curve, threshold, dash-dot curve. Lower plot: linear RMS detector output, solid curve, threshold, dashed line. Threshold set to 1 Vpeak. RMS detector time constant 35 ms, compression ratio 20:1. Input signal level: 6 dB below threshold (-100 ms, 0 ms), 6 dB above threshold (0 ms, 150 ms), 3 dB above threshold (150 ms, 400 ms) and 6 dB below threshold (400 ms, 500 ms).